Section 7-4, Mathematics 104

Complex Fractions

These examples will get progressively more complicated

Example:

$$\frac{\left(\frac{5}{14}\right)}{\left(\frac{25}{8}\right)} = \frac{5}{14} \cdot \frac{8}{25} = \frac{\cancel{5} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot 7 \cdot \cancel{5} \cdot 5} = \frac{4}{35}$$

Example:

$$\frac{\left(\frac{4y^3}{(5x)^2}\right)}{\left(\frac{(2y)^2}{10x^3}\right)} = \frac{4y^3}{(5x)^2} \cdot \frac{10x^3}{(2y)^2} = \frac{4y^3 \cdot 10x^3}{25x^2 \cdot 4y^2} = \frac{40xy}{100} = \frac{2xy}{5}$$

Note: $x \neq 0$ and $y \neq 0$

Example:

$$\frac{\left(\frac{x+1}{x+2}\right)}{\left(\frac{x+1}{x+5}\right)} = \frac{\cancel{x+1}}{x+2} \cdot \frac{x+5}{\cancel{x+1}} = \frac{x+1}{x+2}$$

Note: $x \neq -1$ and $x \neq -5$

Example:

$$\frac{\left(\frac{x^2+4x+3}{x-2}\right)}{2x+6}=\frac{\left(\frac{x^2+4x+3}{x-2}\right)}{\left(\frac{2x+6}{1}\right)}=\frac{x^2+4x+3}{x-2}\cdot\frac{1}{2x+6}=$$

$$\frac{\left(\cancel{x+3}\right)(x+1)}{x-2}\cdot\frac{1}{2\left(\cancel{x+3}\right)}=\frac{x+1}{2(x-2)}$$

Note: $x \neq -3$

Example:

$$\frac{\left(\frac{2}{x+2}\right)}{\left(\frac{3}{x+2}+\frac{2}{x}\right)}=\frac{\left(\frac{2}{x+2}\right)}{\left(\frac{3x}{(x+2)x}+\frac{2(x+2)}{(x+2)x}\right)}=\frac{\left(\frac{2}{x+2}\right)}{\left(\frac{5x+4}{(x+2)x}\right)}=$$

$$\frac{2}{\cancel{x+2}}\cdot\frac{\left(\cancel{x+2}\right)x}{5x+4}=\frac{2x}{5x+4}$$

Note: $x \neq 0$ and $x \neq -1$

Example:

$$\frac{5+x^{-2}}{8x^{-1}+x}=\frac{\left(5+\frac{1}{x^2}\right)}{\left(\frac{8}{x}+x\right)}=\frac{\left(\frac{5x^2+1}{x^2}\right)}{\left(\frac{8+x^2}{x}\right)}=$$

$$\frac{5x^2+1}{x^2}\cdot\frac{x}{8+x^2}=\frac{5x^2+1}{x}\cdot\frac{1}{8+x^2}=\frac{5x^2+1}{x\left(x^2+8\right)}$$

Solving Rational Equations

Example:

$$\frac{x}{2}+1=\frac{3}{5}$$

It's easiest to first get rid of the fractions each side by multiplying by 10

$$10\left(\frac{x}{2}+1\right)=10\left(\frac{3}{5}\right)$$
$$5x+10=6$$
$$5x=-4$$
$$x=-\frac{4}{5}$$

With a variable in the denominator, use the same strategy but keep in mind any values not in the domain cannot be solutions.

$$\frac{8}{3}=\frac{7}{x}-\frac{1}{3x}$$
$$3x\left(\frac{8}{3}\right)=3x\left(\frac{7}{x}-\frac{1}{3x}\right)$$
$$8x=21-1$$
$$8x=20$$
$$x=\frac{5}{2}$$

Note that zero could not be a solution

Example:

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$(x-2)\left(\frac{5x}{x-2}\right) = (x-2)\left(7 + \frac{10}{x-2}\right)$$

$$5x = 7x - 14 + 10$$

$$-2x = -4$$

$$x = 2$$

But 2 is not in the domain of two of the rational expressions so it cannot be a solution. So this equation has no solutions.

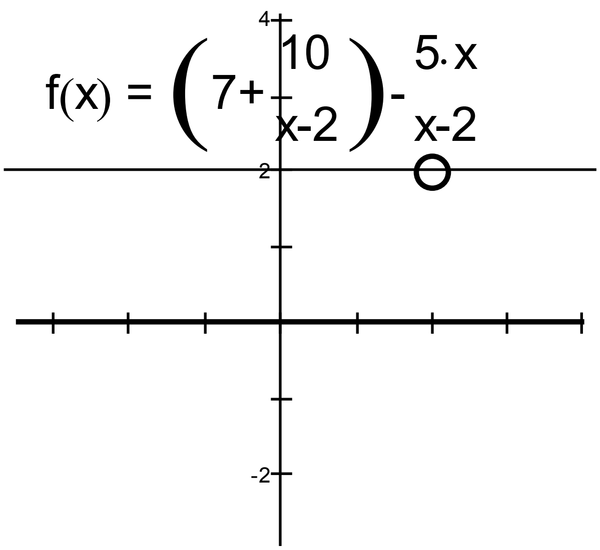
It might be useful to rewrite this as follows:

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$0 = 7 + \frac{10}{x-2} - \frac{5x}{x-2}$$

$$y = 7 + \frac{10}{x-2} - \frac{5x}{x-2}$$

And graph it



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Notice that this function is equal to two every in it's domain.
It just is not defined at x=2.

Using Cross Multiplying

Cross multiplying is a technique to simplify an equation with two fractions.
Here's why it works:

$$\frac{a}{b} = \frac{c}{d}$$

Multiply both sides by *bd*

$$\cancel{b}d\,\frac{a}{\cancel{b}} = b\cancel{d}\,\frac{c}{\cancel{d}}$$
$$ad = bc$$

Which is the result you get by cross multiplying.

Example:

$$\frac{2x}{x+4} = \frac{3}{x-1}$$
$$2x(x-1) = 3(x+4)$$
$$2x^2 - 2x = 3x + 12$$
$$2x^2 - 5x - 12 = 0$$
$$(2x+3)(x-4) = 0$$
$$x = 4$$
$$x = -\frac{3}{2}$$